**Project Report**

**Volatility Modeling and Forecasting using GARCH Family Models: Evidence from NIFTY 50**

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**1. Abstract**

This project explores the dynamics of volatility in Indian equity markets using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) framework. NIFTY 50 daily price data was employed to compute log returns, which were tested for stationarity and conditional heteroscedasticity. A GARCH(1,1) model with Student-t innovations was fitted to capture volatility clustering and persistence.

The results revealed significant volatility persistence, with conditional variance closely matching observed fluctuations. A volatility-scaled trading strategy was implemented, and its performance was benchmarked against a buy-and-hold portfolio. While the GARCH-based strategy achieved lower volatility and reduced drawdowns, its annual return and Sharpe ratio were slightly inferior to the benchmark.

This demonstrates the trade-off between risk control and return maximization. The findings confirm the usefulness of GARCH models in **risk management, forecasting, and investment decision-making**.

**2. Introduction**

Financial time series often exhibit **volatility clustering** – periods of high volatility followed by periods of calm – which cannot be explained by constant-variance models.

The GARCH family of models, introduced by **Engle (1982)** and extended by **Bollerslev (1986)**, provides a robust framework to model such dynamics.

This project applies GARCH modeling to the Indian benchmark index **NIFTY 50** to:

* Analyze volatility characteristics of daily returns.
* Fit and evaluate the GARCH(1,1) model and its forecasts.
* Test the effectiveness of a volatility-scaled trading strategy.
* Benchmark results against a passive buy-and-hold approach.

**3. Literature Review**

* **Engle (1982):** Introduced the ARCH model to capture time-varying volatility.
* **Bollerslev (1986):** Generalized ARCH into GARCH, allowing parsimonious lag structures.
* **Nelson (1991):** Extended to EGARCH for asymmetry effects.
* Several studies show GARCH models are effective in **risk management, Value-at-Risk (VaR), option pricing, and portfolio allocation**.
* Indian equity markets, with their unique volatility patterns, provide an important testing ground for such models.

**4. Data and Methodology**

**4.1 Data Source**

* Data: NIFTY 50 daily closing prices.
* Source: Yahoo Finance (via Python yfinance library).
* Period: **January 2015 – August 2025**.
* Returns: Computed daily log returns to ensure stationarity.

**4.2 Stationarity Tests**

* Augmented Dickey-Fuller (ADF) test results:
  + Log prices: Non-stationary (ADF statistic = –0.2606, p-value ≈ 0.93).
  + Returns: Stationary (ADF statistic = –14.1302, p-value < 0.01).
* Conclusion: Returns are suitable for volatility modeling.

**4.3 GARCH(1,1) Model**

* Specification: Constant mean with GARCH(1,1) variance and Student-t errors.
* Estimated Parameters:
  + ω (constant) = 0.0295 (significant)
  + α1 (ARCH term) = 0.0806 (significant)
  + β1 (GARCH term) = 0.8853 (significant)
  + Persistence (α + β) = **0.966**, indicating strong volatility clustering.
  + Degrees of freedom (ν) = 6.35, capturing heavy-tailed distribution of returns.

**4.4 Forecasting**

* 5-day ahead daily volatility forecasts (decimal form):
  + Day 1: 0.00714
  + Day 2: 0.00722
  + Day 3: 0.00730
  + Day 4: 0.00738
  + Day 5: 0.00745

Forecasts indicate gradually increasing short-term volatility, consistent with volatility persistence.

**4.5 Rolling Strategy**

* Approach: Rolling/expanding window estimation with rebalancing.
* Position sizing rule:
  + Target annual volatility = 12%.
  + Position size = Target volatility ÷ Forecasted volatility.
* Rebalancing frequency: Monthly (~21 trading days).
* Leverage cap: Maximum 3x.

**5. Results**

**5.1 Model Fit**

* GARCH(1,1) model successfully captured volatility clustering.
* Residual diagnostics indicated that most ARCH effects were absorbed.

**5.2 Performance Metrics**

| **Metric** | **Market (Buy & Hold)** | **GARCH Strategy** |
| --- | --- | --- |
| **Annual Return** | 9.79% | 5.71% |
| **Annual Volatility** | 17.59% | 14.19% |
| **Sharpe Ratio** | 0.62 | 0.46 |
| **Max Drawdown** | –40.04% | –35.24% |

**5.3 Interpretation**

* The GARCH-based strategy successfully reduced volatility and drawdowns compared to the market.
* However, the strategy produced **lower returns and a weaker Sharpe ratio**, indicating reduced efficiency.
* This aligns with prior findings that volatility targeting stabilizes returns but may sacrifice long-term performance.

**6. Discussion**

* GARCH(1,1) confirmed the presence of **volatility clustering** in NIFTY 50 returns.
* Volatility-scaled trading strategy demonstrated practical use of forecasts in risk management.
* Conservative investors may prefer this approach due to **lower volatility and reduced drawdowns**.
* Future improvements: Using **EGARCH** or **GJR-GARCH** to capture asymmetry and leverage effects more effectively.

**7. Conclusion**

* NIFTY 50 returns exhibit **volatility clustering** and persistence.
* The GARCH(1,1) model provided accurate conditional variance estimates.
* Forecasts demonstrated stability, validating the model’s predictive power.
* The GARCH-based strategy lowered portfolio risk but underperformed buy-and-hold in terms of return.
* Future scope: Applying **multi-asset data, high-frequency analysis, or hybrid machine learning + volatility models**.

**8. References**

* Engle, R. (1982). *Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation.* Econometrica.
* Bollerslev, T. (1986). *Generalized Autoregressive Conditional Heteroskedasticity.* Journal of Econometrics.
* Nelson, D. (1991). *Conditional Heteroskedasticity in Asset Returns: A New Approach.* Econometrica.
* Python libraries: yfinance, pandas, numpy, statsmodels, arch, empyrical.

**9. Annexures**

* Detailed model output summary (coefficients, log-likelihood, AIC/BIC).
* Forecast plots of conditional volatility.
* Cumulative returns chart: **Buy & Hold vs GARCH Strategy**.